



Measuring the signal-to-noise ratio in magnetic resonance imaging: a caveat[☆]

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Abstract

The validity of the signal-to-noise ratio (SNR) as an objective quality measure for biomedical images has been the subject of a long-standing debate. Nevertheless, the SNR is the most popularly used measure both for assessing the quality of images and for evaluating the effectiveness of image enhancement and signal processing techniques. In this correspondence, we illustrate that under certain conditions the SNR can be changed by a nonlinear transformation, and also that it is often hard to *measure* objectively. Therefore, the issue is not only how well the SNR correlates with image quality as perceived by a human observer (which has been the primary subject of earlier debate), but also that SNR is questionable from a quantitative measurement point of view.

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1. Introduction

Magnetic resonance imaging (MRI) is a notable medical imaging technique that has proven to be particularly valuable for examination of the soft tissues in the body (such as the brain), and it has become an instrumental tool for the diagnosis of stroke and other significant diseases as well as for pinpointing the focus of diseases such as epilepsy [6]. It is also considered to be an extremely important instrument

for the study of other parts of the nervous system (such as the spinal cord), as well as various joints, the thorax, the pelvis and the abdomen. Because of the recent interest in signal processing for improving the image quality, which is often quantified by the estimated signal-to-noise ratio (SNR), it is imperative to understand how this measure can be affected by nonlinear signal processing operations.¹

Most MR imaging scenarios are limited by the SNR in the reconstructed image. In particular, although it is

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¹ Although the physical ratio-of-amplitudes is commonly used to quantify SNR in the MRI literature, in this paper, we will assume the engineering convention of ratio-of-powers. There is a square-relationship between the former and the latter.

sometimes argued that improving SNR beyond 20 dB is diagnostically not significant for static imaging (see, e.g., [3,7,15]), *an improvement in SNR can always be translated into an increase in acquisition speed* and therefore be used to reduce the imaging cost and motion artifacts. Therefore, improving the SNR in MR images has become extremely critical for reducing motion artifacts and in applications where imaging speed is a major concern. Such applications include imaging of dynamic processes, such as the heart [13]. Also, since an improvement in SNR can significantly cut imaging times, it can increase the cost-effectiveness of MRI equipment in a hospital environment as well as decrease breath-holding durations and other discomforts for patients.

Evaluation of the quality of a real-world image is often a subjective task, and perhaps due to the absence of more sophisticated indicators, the SNR appears to be one of the most popularly used measures of the quality of an MR image. In general the SNR does not measure bias errors (which are often significant), and furthermore there is not always a clear correlation between the SNR and the image quality as visually perceived by a human observer, which is more related to the contrast in a broad sense (see, e.g., [14, Chapter 7] for a discussion of visual image quality).

In this communication we illustrate how SNR can be manipulated by nonlinear operations on the signals, and that it is sometimes also difficult to measure objectively. The goal of this paper is, therefore, to emphasize the fact that caution should be exercised when the SNR measured from an MR image is used as a quality measure, or as an indicator for the improvement offered by a signal processing algorithm that possibly employs a nonlinear operation at some stage of processing. Examples of such algorithms include image enhancement procedures using median filters or nonlinear anisotropic filtering techniques (see, e.g., [4,16]).

In this study, we assume that a real-valued MR image is already obtained from the raw k -space data and that necessary corrections to reduce phase distortions may have been applied as discussed in [8,9,12]. However, since the results on the distortion of SNR under nonlinear operations are true in general, similar effects are expected to occur if nonlinear techniques are employed when reconstructing images from the raw k -space data.

2. The SNR

As we illustrate in this section, the major drawback of the SNR as a quality measure is that it is not invariant to nonlinear transformations. Consider an observation model of the form

$$x = s + e, \quad (1)$$

where s is a signal of interest, and e is noise. We assume that both s and e are random variables. Also, throughout this paper we assume for simplicity that all signals and noise are real valued,² and that the noise is zero mean.³ The SNR in x is given by

$$\text{SNR}_x = \frac{E\{s^2\}}{E\{e^2\}}, \quad (2)$$

where $E\{\cdot\}$ stands for statistical expectation.

Let us consider the following nonlinear transformation of x :

$$y = f(x) = f(s + e) = f(s) + \sum_{k=1}^{\infty} \frac{f^{(k)}(s)e^k}{k!}, \quad (3)$$

where $k!$ is the factorial of k and $f^{(k)}(x)$ is the k th derivative of $f(x)$, assuming that all derivatives of $f(x)$ are well defined. The SNR in y is equal to

$$\begin{aligned} \text{SNR}_y &= \frac{E\{f^2(s)\}}{E\{(\sum_{k=1}^{\infty} f^{(k)}(s)e^k/k!)^2\}} \\ &\approx \frac{E\{f^2(s)\}}{E\{f'^2(s)\}E\{e^2\}}, \end{aligned} \quad (4)$$

where by convention $f'(s) = f^{(1)}(s)$ and $f'^2(s) = (f'(s))^2$, and where the approximation is valid when $\text{SNR}_x \gg 1$. We conclude that $\text{SNR}_y > \text{SNR}_x$ exactly when

$$\frac{E\{f^2(s)\}}{E\{f'^2(s)\}} > E\{s^2\} \quad (5)$$

and therefore nonlinear transformations can improve the SNR in a signal, provided that the function $f(x)$ and the statistical distribution of s are such that (5) holds.

² All results extend to the complex case as well.

³ Note that if *magnitude images* are considered, the noise is not zero mean, but the analysis here could be extended to such cases. See, e.g., [2,5,12] for more discussion on the statistics of the noise in MR images.

In general, the conditions on $f(x)$, under which (5) holds, depend on the distribution of s . However, we can easily study a few special cases. If, for example, $f(x) = x^2$, then $f'(x) = 2x$ and hence

$$\frac{\text{SNR}_y}{\text{SNR}_x} = \frac{E\{f(s)^2\}}{E\{f'(s)\}E\{e^2\}} \frac{E\{e^2\}}{E\{s^2\}} = \frac{E\{s^4\}}{4(E\{s^2\})^2}. \quad (6)$$

We conclude that $\text{SNR}_y > \text{SNR}_x$ if and only if

$$E\{s^4\} > 4(E\{s^2\})^2. \quad (7)$$

For zero-mean random signals s , (7) holds exactly when

$$\kappa(s) = \frac{E\{s^4\} - 3(E\{s^2\})^2}{(E\{s^2\})^2} > 1, \quad (8)$$

where $\kappa(s)$ is the *Kurtosis* of s . (For a Gaussian distribution, $\kappa(s) = 0$; distributions for which $\kappa(s) > 0$ are called *super-Gaussian*, and distributions for which $\kappa(s) < 0$ are called *sub-Gaussian*.) This means that if the probability distribution of the image is highly super-Gaussian (most natural images are in this class), i.e., denser around the mean and heavier at the tails (e.g., Laplacian) then the square-operation (such as the one used in creating magnitude images) could deceptively demonstrate an improvement in SNR.

An interesting question is whether it is possible to find a function $f(x)$ such that $\text{SNR}_y > \text{SNR}_x$ regardless of the distribution of s . Without loss of generality, consider a unit-power signal s (i.e., $E\{s^2\} = 1$). Then from (5) $\text{SNR}_y > \text{SNR}_x$ if $f^2(s) > f'^2(s)$ for all s . This can be achieved if we choose $f(s) = a^s$ where a satisfies $1/e < |a| < e$, because in that case $|\ln |a|| < 1$ which implies that $f^2(s) = a^{2s} > (\ln |a|)^2 a^{2s} = f'^2(s)$. Hence, a nonlinearity in the form of an exponential function with exponent a in the range $1/e < |a| < e$ will improve the SNR for any s with unit power. An interesting consequence of this result concerns log-magnitude images. In some cases, to improve the contrast and the dynamic range, a logarithmic nonlinearity might be applied to the constructed image. According to the above analysis, we conclude that the original image (in linear scale) will have a higher SNR than the log-scale image, although the contrast of the latter is significantly better especially for low-signal power regions. This demonstrates how insufficient SNR is in representing the visual clues that human perception looks for when assessing image quality.

3. Measuring the SNR

In the previous section we have seen that for signals with certain properties, a nonlinear transformation can change the SNR. Next, we discuss the difficulties associated with *measuring* the SNR (see also [10]). Let us, for simplicity consider a signal consisting of two regions, one area Ω_s with N_s samples of a signal $\{s_n\}$ of interest, and one region Ω_n consisting of N_n samples $\{e_n\}$ that are known to be pure zero-mean noise, and which are independent of the signal. Hence, the signal observed at a pixel n can be written as

$$x_n = \begin{cases} s_n + e_n, & n \in \Omega_s, \\ e_n, & n \in \Omega_n. \end{cases} \quad (9)$$

The SNR in $\{x_n\}$ is $\text{SNR}_x = E\{s_n^2\}/E\{e_n^2\}$.

For a given image, the SNR is usually estimated by using a moment-based estimator of the form

$$\widehat{\text{SNR}}_x = \frac{(1/N_s) \sum_{n \in \Omega_s} x_n^2}{(1/N_n) \sum_{n \in \Omega_n} x_n^2}, \quad (10)$$

where N_s and N_n are the numbers of pixels in the signal and the noise region, respectively. For a reasonably high SNR_x and for a large number of measured pixels, we have that

$$\begin{aligned} \widehat{\text{SNR}}_x &= \frac{(1/N_s) \sum_{n \in \Omega_s} x_n^2}{(1/N_n) \sum_{n \in \Omega_n} x_n^2} \approx \frac{E\{(s_n + e_n)^2\}}{E\{e_n^2\}} \\ &= \frac{E\{s_n^2\} + E\{e_n^2\}}{E\{e_n^2\}} = \text{SNR}_x + 1 \approx \text{SNR}_x, \end{aligned} \quad (11)$$

where we used the assumption that the noise e_n has zero mean and is independent of s_n (this equation was discussed in more detail by Henkelman [8]). Note from (11) that the measured SNR is always larger than the true SNR. However, when SNR_x is high, estimating it via (10) in general gives reliable results.

We next discuss how the measured SNR can change when the signal $\{x_n\}$ is transformed via a quadratic nonlinear function.⁴ For illustration purposes, we assume that s is constant (i.e., $s_n = s$ is a deterministic

⁴ The square-nonlinearity is assumed due to its simplicity and its common occurrence in signal processing techniques and magnitude operations. However, similar analyses could be carried out for other possible types of nonlinear operations encountered in the processing.

quantity) throughout Ω_s , and that we form a new signal $\{y_n\}$ according to

$$y_n = x_n^2. \tag{12}$$

From the analysis in Section 2 we know that for a constant signal, $\text{SNR}_y \approx \text{SNR}_x/4$ and hence the SNR in $\{y_n\}$ is less than that in $\{x_n\}$. (This is natural since the sign, or the phase for complex data, is lost when the transformation (12) is applied.) Nevertheless, the SNR in $\{y_n\}$, as *measured* via (10) can be much larger than the SNR measured from the original image $\{x_n\}$. To understand why this is so, consider the measured SNR in $\{y_n\}$, assuming that $\text{SNR}_x \gg 1$:

$$\begin{aligned} \widehat{\text{SNR}}_y &= \frac{(1/N_s) \sum_{n \in \Omega_s} y_n^2}{(1/N_n) \sum_{n \in \Omega_n} y_n^2} \\ &= \frac{(1/N_s) \sum_{n \in \Omega_s} [s^2 + 2se_n + e_n^2]^2}{(1/N_n) \sum_{n \in \Omega_n} e_n^4} \\ &\approx \frac{(1/N_s) \sum_{n \in \Omega_s} s^4}{(1/N_n) \sum_{n \in \Omega_n} e_n^4}. \end{aligned} \tag{13}$$

This expression essentially behaves as $(\widehat{\text{SNR}}_x)^2$. Therefore, we expect the measured SNR in $\{y_n\}$ to be much larger than it actually is; i.e., the squaring in (12) makes the signal appear to an observer as if it were much less noisy.

4. Illustration

We provide two examples to illustrate the phenomena discussed in the previous sections. In the first example, we consider a simulated constant signal embedded in zero-mean Gaussian noise. In the second example, we use real MRI data from a cat spinal cord.

Example 1 (Step function in noise). We consider a signal consisting of two segments Ω_s and Ω_n , during which the signal level is equal to 10 and 0, respectively, embedded in white Gaussian noise with variance σ^2 . The signal along with its noisy version are shown in Fig. 1(a) for $\sigma^2 = 1$. In Fig. 1(b) we show the signal after the nonlinear transformation (12). Finally, in Fig. 1(c) we show the true SNR for the original signal, the measured SNR for the original signal (as defined via (10)), the true SNR for the transformed

signal, and the measured SNR in the transformed signal (as defined via (13)), for some different values of $1/\sigma^2$. The true SNR in the transformed signal y_n is approximately 6 dB lower than the SNR in the original signal x_n , when $1/\sigma^2$ is high. (We can see that the measured SNR of x_n converges to the true SNR in this case; cf. (11).) This 6 dB difference in SNR between y_n^2 and x_n^2 corresponds to the theoretical value of $\frac{1}{4}$ described in Section 2. On the other hand, the measured SNR in $\{y_n\}$ appears much larger than the true SNR, which corroborates the findings of Section 3.

Example 2 (Cat spinal cord). We analyze data from a cat spinal cord using a 4.7 T MRI scanner (obtained with TR = 1000 ms, TE = 15 ms, FOV = 10 × 5 cm, matrix = 120 × 120, slice thickness = 2 mm, sweep width = 26 khz, 1 average) [1]. The data collected from a phased array of four coils is combined using the sum-of-squares (SoS) technique to yield a reconstructed image.⁵ Let y_k be the observed pixel value from coil k :

$$y_k = \rho c_k + n_k, \quad k = 1, 2, 3, 4, \tag{14}$$

where ρ is the (real-valued) object density (viz. the MR contrast), c_k is the (complex-valued) sensitivity associated with coil k for the image voxel under consideration, and n_k is zero-mean complex-valued noise. The SoS reconstruction for this voxel is obtained via⁶

$$\hat{\rho} = \sqrt{\sum_{k=1}^4 |y_k|^2}. \tag{15}$$

We consider two different nonlinear operations on the SoS reconstruction: natural logarithm and median filtering (MF). The former nonlinear operation simply generates a new image by modifying the pixel-by-pixel values by applying the *log* function. The latter one is a standard nonlinear image processing technique that is robust to outliers, which is often used to improve SNR. In median filtering, each pixel value is simply replaced by the median of the values of its neighboring pixels (here we use a 5 × 5 region centered at the pixel of interest).

⁵ It is known that the SNR of the SoS reconstructed image asymptotically approaches that of the best linear unbiased reconstruction image [11].

⁶ The image voxel value estimate $\hat{\rho}$ corresponds to the signal x in the arguments presented in the previous sections.

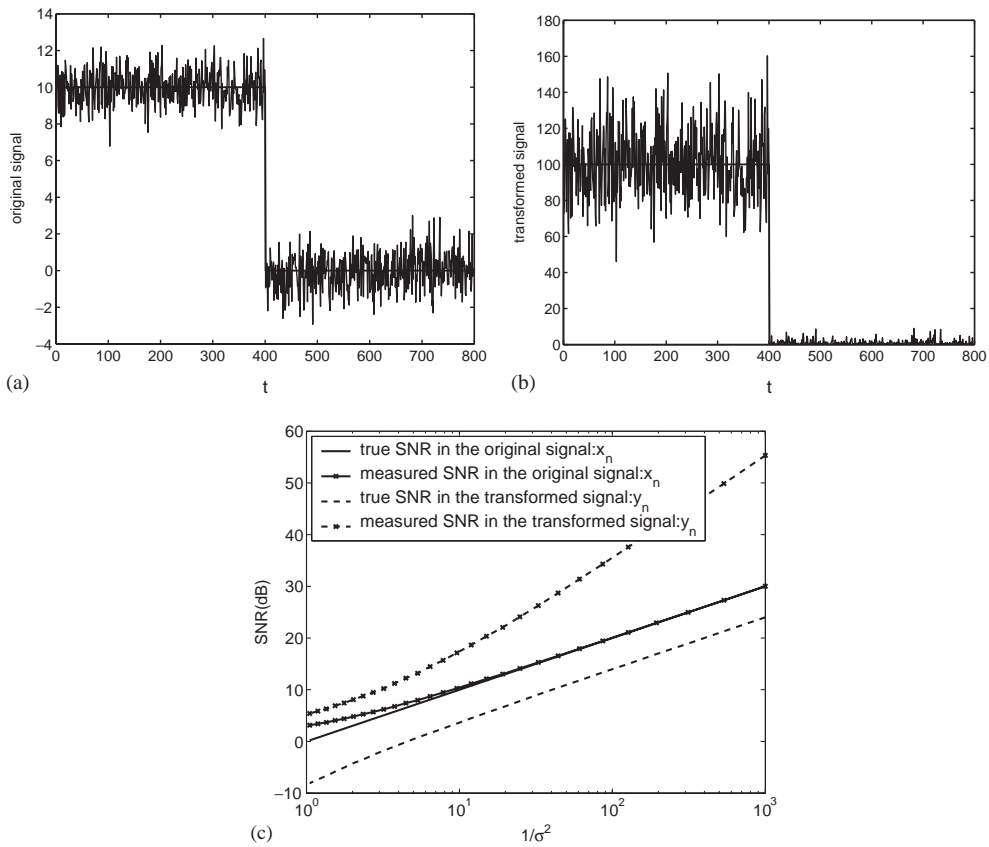


Fig. 1. Synthetic data example. (a) Original noisy step function signal x_n , (b) transformed (squared) signal y_n , and (c) the true and the measured SNR levels.

In Fig. 2 we present the images provided by SoS, log-SoS, and MF-SoS as well as their corresponding local SNR estimates using the reference noise region shown in the upper-right corner. Observe that although the MF-SoS image exhibits an improved SNR compared to the original SoS, the image actually *looks worse*. On the other hand, the SNR of the log-SoS image decreased, yet the dynamic range and the signal contrast in low-power regions are improved.

5. Concluding remarks

SNR is a popular measure of quality in phased-array MRI reconstruction. Nevertheless, it is hard to measure objectively in a practical set-up. It is also easy to manipulate, because nonlinear transformations can

make the SNR appear higher or lower in a manner that is uncorrelated with the perceived image quality. Therefore the SNR must be used with careful judgement as a quality measure when evaluating images processed by certain signal processing procedures. In this communication we have illustrated how general nonlinear operations on the (magnitude) images could alter the experimentally measured SNR. An especially alarming observation is that for certain nonlinear operations the true SNR increases, whereas the experimentally measured SNR decreases (and vice versa). Another problem that observed is that the SNR may be improved at the cost of a decrease in perceived quality.

Our observations call for an extended debate on objective quality measures that are invariant (at least to some extent) to nonlinear operations on the signals.

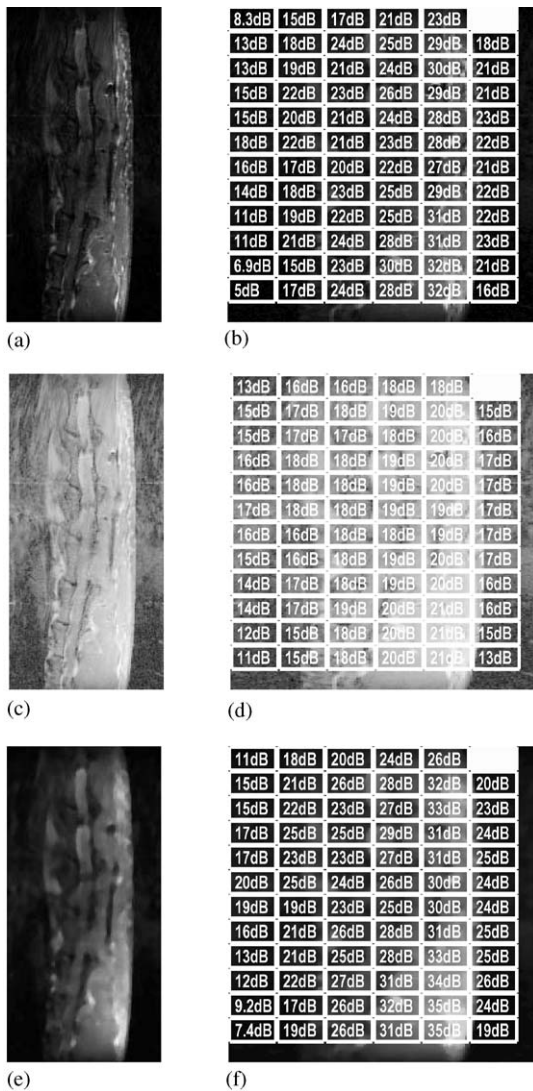


Fig. 2. Reconstruction images and their SNR performance. (a) SoS, (b) SNR of SoS, (c) logarithm of SoS, (d) SNR of logarithm of SoS, (e) median filtered SoS, (f) SNR of median filtered SoS.

Although we do not propose such a new quality measure here, we should note that one possible class of such measures here, we should note that one possible class of such measures includes quantities derived from information theory (see, e.g., [17] for ideas along these lines).

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